► KOHTARO TADAKI, *Fixed point theorems on partial randomness.*
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In algorithmic information theory, the program-size complexity $H(s)$ of a finite binary string s is defined as the length of the shortest binary program for the universal self-delimiting Turing machine U to output s, and plays a crucial role in characterizing the randomness of a real. As an example of random real, Chaitin introduced the halting probability Ω in [1]. In [2] we generalized Ω to $\Omega(T)$ by $\Omega(T) = \sum 2^{-|p|/T}$, where the sum is over all programs p for U, so that the partial randomness of $\Omega(T)$ can be controlled by a real $T \in (0, 1]$.

Recently, we showed that the computability of $\Omega(T)$ gives a sufficient condition for T to be a fixed point on partial randomness, as follows.

THEOREM 1 ([3]). *For every $T \in (0, 1)$, if $\Omega(T)$ is computable, then $Tn \leq H(T_n) + O(1)$ and $H(T_n) \leq Tn + o(n)$, and therefore $\lim_{n\to\infty} H(T_n)/n = T$, where T_n is the first n bits of the base-two expansion of T.*

Theorem 1 was obtained in [3] on developing a statistical mechanical interpretation of algorithmic information theory, where $\Omega(T)$ is interpreted as a partition function of a statistical mechanical system. In the interpretation, we also introduced the notions of thermodynamic quantities; free energy $F(T)$, energy $E(T)$, and statistical mechanical entropy $S(T)$, which are variants of $\Omega(T)$.

In this talk, we show that Theorem 1 holds for each of all these quantities, instead of $\Omega(T)$. In particular, since $F(T) = -T \log_2 \Omega(T)$, we see that the computability of $F(T)$ gives completely different fixed points from the computability of $\Omega(T)$.

[1] G. J. CHAITIN, *A theory of program size formally identical to information theory*, ***Journal of the ACM***, vol. 22 (1975), no. 3, pp. 329–340.

[2] K. TADAKI, *A generalization of Chaitin's halting probability Ω and halting self-similar sets*, ***Hokkaido Mathematical Journal***, vol. 31 (2002), no. 1, pp. 219–253.

[3] ———, *A statistical mechanical interpretation of algorithmic information theory*, ***Proceedings of Computability in Europe 2008*** (University of Athens, Greece), 2008, to appear.